

CONSIDERATIONS ON THE EXCITATION MECHANISM OF THE VERTICAL VIBRATION SYMMETRICAL AND ANTISYMMETRICAL MODES IN RAILWAY VEHICLES

MĂDĂLINA DUMITRIU¹

Abstract: The railway vehicles are complex oscillator systems featuring specific vibration characteristics. The symmetrical and antisymmetrical vibration modes represent a property of the railway vehicle vibrations, due to its symmetry in construction. The paper describes the excitation mechanism of the symmetrical and antisymmetrical modes in a vertical plan of a railway vehicle on a track with irregularities. The symmetrical and antisymmetrical modes of the axles, as well as the vibration modes of the vehicle, are focused on by an appropriate processing the vehicle movement equations.

Key words: railway vehicles, track irregularity, vertical vibration, symmetrical/antisymmetrical modes, bounce-pitch-forward modes

1. INTRODUCTION

The railway vehicle represents a complex oscillator system, subjected to a mode of vibrations with specific characteristics. Thus, the railway vehicle oscillating movements are established in both horizontal and vertical plans but the two types of vibrations are decoupled, due to the construction symmetries (inertial, elastic and geometrical) [1]. The oscillating movements are found as independent movements or coupled between them, which represent a disturbance from the steady/state motion of the vehicle along the track. Such oscillations are made up of the simple vibration modes of the vehicles' suspended masses – the rigid modes and the complex modes of vibration [2].

As a construction particularity in the railway vehicle, which generally complies with the symmetry rules, the vibration modes of the vehicle' suspended masses are symmetrical and antisymmetrical.

¹ *PhD. Stud. Eng. Politehnica University of Bucharest, madalinadumitriu@yahoo.com*

The major cause of vibration in the railway vehicles lies in the interaction with the rolling track. In its construction, the rolling track has a series of deviations from the ideal geometrical shape, on the one hand, and there are also defects of the rails' rolling track and discontinuities in their structure, on the other hand. All these irregularities of the rolling track constitute themselves in excitation factors of the railway vehicles vibrations [3].

In a vertical plan, the irregularities of the rolling track transfer via the elastic contact of the rolling surfaces to the axles, thus generating translation movements (bounce) and rotation (pitch) of their plans [4]. Moreover, such movements are conveyed via the suspension elements at the vehicle' suspended masses and generate their simple vibration modes, the bounce and pitch, as well as the complex modes of vibration, global or local, due to the carbody elasticity characteristics [5].

The paper herein describes the railway vehicle excitation mechanism of the symmetrical and antisymmetrical modes of vibration in a vertical plan. The symmetrical and antisymmetrical movements of bounce and pitch are presented for the axles plans coming from the rolling track irregularities, movements that are conveyed by the suspension elements of the vehicle suspended masses, thus developing their corresponding symmetrical and antisymmetrical vibration modes – the bogies bounce and pitch, the carbody bounce, pitch and bending. Similarly, there is a description of the mechanism that helps with the excitation of the rebound movement of the bogies and axles plans, due to the carbody bending vibrations.

The symmetrical and antisymmetrical modes of the axles plans, as well as the corresponding vehicle vibration modes are focused on via an appropriate processing of the vehicle movement equations that correspond to the circulation on an irregular track. To this purpose, while adopting the hypothesis of a perfectly rigid track, the vehicle-track system is reduced to an equivalent mechanical model, with 18 degrees of freedom, where the vehicle model includes a body with parameters distributed for carbody and a system of rigid bodies, namely the axles and the suspended masses of the two bogies, plus a set of Kelvin-Voigt systems that models the vehicle's suspension stages, the transmission system of the longitudinal force and the system of the axles' elastic steering.

2. MECHANICAL MODEL OF THE VEHICLE-ROLLING TRACK SYSTEM

The case of a four-axle vehicle, two-suspension stages, which travels at a constant velocity on a track with longitudinal irregularities is being presented. The vehicle-track system is reduced to a mechanical model, equivalent with 18 degrees of freedom, and the model parameters are explained in Table 1.

For the vehicle carbody, modelled via an Euler-Bernoulli beam, there are considered the first two natural modes of bending in a vertical plan (symmetrical and antisymmetrical), as well as the rigid modes of vibration, namely the bounce z_c and pitch θ_c .

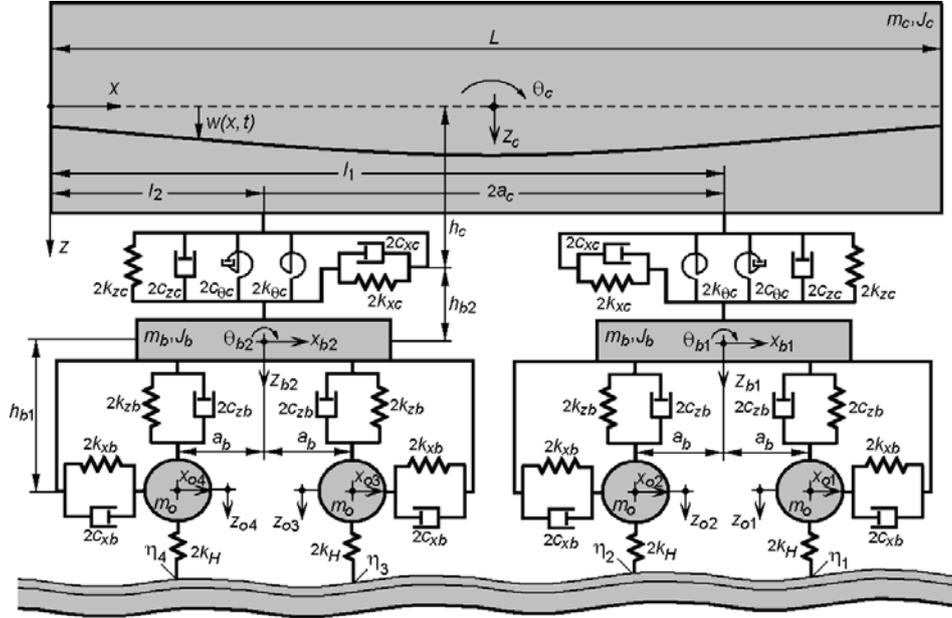


Fig. 1. The mechanical model of the vehicle-rolling track system.

The vertical displacement of the carbody $w(x, t)$ is given by the overlapping of the rigid modes of vibration with the bending ones [6, 7]

$$w(x, t) = z_c(t) + \left(x - \frac{L}{2}\right)\theta_c(t) + \sum_{n=2}^3 X_n(x)T_n(t), \quad (1)$$

where $X_n(x)$, with $n = 2, 3$, are the eigenfunctions of the first two modes of the carbody vertical bending and $T_n(t)$ is the time coordinate of the natural mode of bending n .

The suspended masses of the bogies are considered three-degree freedom rigid bodies, with the following movements: bounce z_{bi} , rebound x_{bi} and pitch θ_{bi} , with $i = 1, 2$. The vehicle axles are also perceived as two-degree freedom rigid bodies that can perform translation movements on the vertical direction $z_{oj,(j+1)}$ and translation movements on the longitudinal direction $x_{oj,(j+1)}$, with $j = 2i-1$, and $i = 1, 2$, where each bogie is equipped with the axles j and $j+1$.

The two suspension stages of the vehicles are modelled via the Kelvin-Voigt systems. The primary suspension is also modelled by two Kelvin-Voigt systems that operate in translation in the vertical and longitudinal directions, while the secondary suspension deals with three Kelvin-Voigt systems – two for the translation (vertical and longitudinal) and one for the rotation. The Kelvin-Voigt system in the longitudinal direction between the carbody and bogie models the system of conveying the longitudinal forces and the Kelvin-Voigt longitudinal system in the plan of axles models their elastic steering.

Table 1. The parameters of the mechanical model of the vehicle-rolling track system.

Carbody mass	m_c
Bogie suspended mass	m_b
Axle mass	m_o
Carbody inertia moment	J_c
Bogie inertia moment	J_b
Carbody length	L
Carbody wheelbase	$2a_c$
Bogie wheelbase	$2a_b$
The carbody bearing points position on secondary suspension	$l_{1,2}$
Distance between the carbody medium fiber to the longitudinal forces transmission system	h_c
Distance between the longitudinal forces transmission system and the center of gravity of the bogie suspended mass	h_{b2}
Distance between the center of gravity of the bogie suspended mass and the axles elastic steering system	h_{b1}
Vertical stiffness of the secondary suspension	$2k_{zc}$
Longitudinal rigidity of the secondary suspension	$2k_{xc}$
Carbody-bogie angle rigidity	$2k_{\theta c}$
Vertical damping of the secondary suspension	$2c_{zc}$
Longitudinal damping of the secondary suspension	$2c_{xc}$
Carbody-bogie angle damping	$2c_{\theta c}$
Vertical rigidity of the primary suspension	$2k_{zb}$
Longitudinal rigidity of the primary suspension	$2k_{xb}$
Vertical damping of the primary suspension	$2c_{zb}$
Longitudinal damping of the primary suspension	$2c_{xb}$
Wheel-rail contact rigidity	$2k_H$

The hypothesis of a perfectly rigid track is being adopted, which forces a certain travelling upon the vehicle due to the track irregularities. The irregularities against the axles are described by the functions $\eta_{j,(j+1)}$, with $j = 2i-1$, for $i = 1, 2$, dependant on the distance along the rolling track [6, 7].

The elasticity of the wheel-rails contact is dealt with by introducing certain elastic elements with a hertzian-type linear characteristic.

3. MECHANISM OF THE SYMMETRICAL AND ANTISYMMETRICAL EXCITATION FOR THE VEHICLE VIBRATION MODES

The irregularities of the rolling rack are transmitted to the axles via the elastic contact of the rolling surfaces, thus generating symmetrical and antisymmetrical movements in the vertical direction of their plans.

In Fig. 2, the travelling of the front bogie axles plan is presented, as coming from the track longitudinal irregularities. The plan of the axles has a translation movement – bounce z (fig. 2, *a*), and a rotation movement – pitch θ (fig. 2, *b*). For the bounce, it can be noticed that the axles position is symmetrical compared to axis OZ

crossing in the middle of distance between them, whereas the axles are in an antisymmetrical position for the pitch.

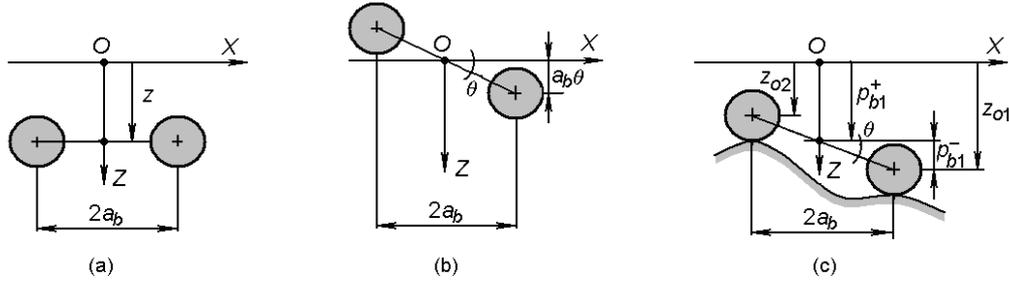


Fig. 2. Bounce and pitch of the axles plan for the front bogie.

The position of each bogie axle compared to the reference OXZ (fig. 1, c), located on the symmetry axis of the axles plan, is the result of the overlapping of the two movements, bounce and pitch, namely

$$z_{o1} = p_{b1}^+ + p_{b1}^-; \quad z_{o2} = p_{b1}^+ - p_{b1}^-, \quad (2)$$

where $p_{b1}^+ = z$ is the travelling of axles plan of the front bogie because of the bounce, and $p_{b1}^- = a_b\theta$ is the travelling coming from the pitch. Further on,

$$p_{b1}^+ = \frac{1}{2}(z_{o1} + z_{o2}); \quad p_{b1}^- = \frac{1}{2}(z_{o1} - z_{o2}). \quad (3)$$

In a similar way and in dependence on the coordinates z_{o3} and z_{o4} which describe the positions of the axles 3 and 4,

$$z_{o3} = p_{b2}^+ + p_{b2}^-; \quad z_{o4} = p_{b2}^+ - p_{b2}^-. \quad (4)$$

the travelling of the axles plan corresponding to the rear bogie is obtained

$$p_{b2}^+ = \frac{1}{2}(z_{o3} + z_{o4}); \quad p_{b2}^- = \frac{1}{2}(z_{o3} - z_{o4}). \quad (5)$$

The combination of the bounce and pitch movements of the axles plans for the two bogies will result into the symmetrical and antisymmetrical movement modes of the vehicle's axles plans, as seen in figures 3 and 4. Figure 3 features the movement of the symmetrical bounce of the vehicle's axles plans, where all four axles are moving in phase. Likewise, the antisymmetrical bounce of the axles plan is visible, where the axles of a bogie are in antiphase with the axles of the other bogie.

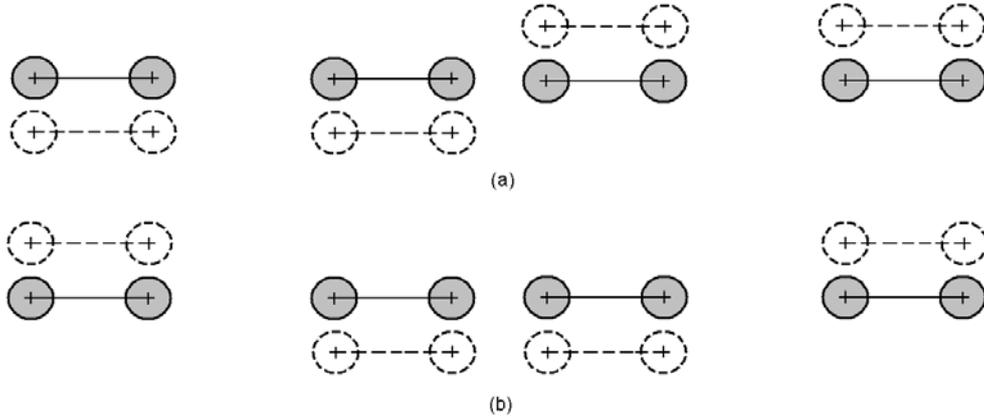


Fig. 3. The bounce of the axles plans: (a) symmetrical bounce; (b) antisymmetrical bounce.

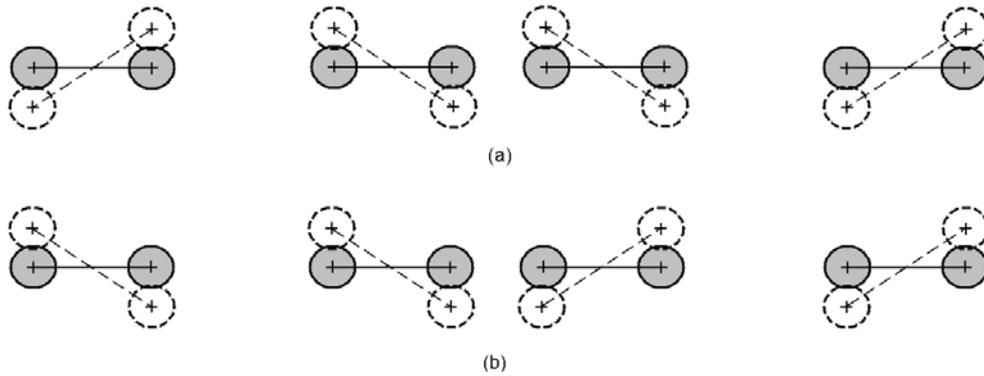


Fig. 4. The pitch of the axles plans: (a) symmetrical pitch; (b) antisymmetrical pitch.

Figure 4 shows the pitch shapes of the vehicle’s axles plans. The symmetrical pitch defines itself by rotations in antiphase of the axles plans at the two bogies, while the antisymmetrical pitch is characterized by rotation in phase movements of the axles plans.

In dependence on the values p_{bi}^+ (with $i = 1, 2$), which define the bounce movements of the axles plans for the two bogies, the parameters for the symmetrical bounce of the vehicle’s axles plans can be defined

$$p_{vs}^+ = \frac{1}{2}(p_{b1}^+ + p_{b2}^+) = \frac{1}{4}(z_{o1} + z_{o2} + z_{o3} + z_{o4}) \quad (6)$$

and for the antisymmetrical bounce,

$$p_{vs}^- = \frac{1}{2}(p_{b1}^+ - p_{b2}^+) = \frac{1}{4}(z_{o1} + z_{o2} - z_{o3} - z_{o4}). \quad (7)$$

Also, as a function of the values p_{bi}^- corresponding to the pitch of the axles plans for the two bogies, the parameters of the pitch movements of the vehicle's axles plans can be defined:

- for the symmetrical pitch

$$p_{vg}^+ = \frac{1}{2}(p_{b1}^- - p_{b2}^-) = \frac{1}{4}(z_{o1} - z_{o2} - z_{o3} + z_{o4}); \quad (8)$$

- for the antisymmetrical pitch

$$p_{vg}^- = \frac{1}{2}(p_{b1}^- + p_{b2}^-) = \frac{1}{4}(z_{o1} - z_{o2} + z_{o3} - z_{o4}). \quad (9)$$

The pitch and bound movements of the axles plans are transmitted by the elements of the primary suspension to the bogies, thus generating corresponding symmetrical and antisymmetrical vibration modes.

Figure 5 presents the modes of symmetrical and antisymmetrical bounce. For the symmetrical bounce, it can be noticed that the two bogies move on the vertical direction, in phase, whereas the two bogies are in antiphase during the antisymmetrical bounce. Figure 6 features the pitch modes of the bogies, which are similar to the axles plans'. The symmetrical pitch implies the rotations of the two bogies to be in antiphase, unlike the antisymmetrical pitch when the bogies are rotated in phase.

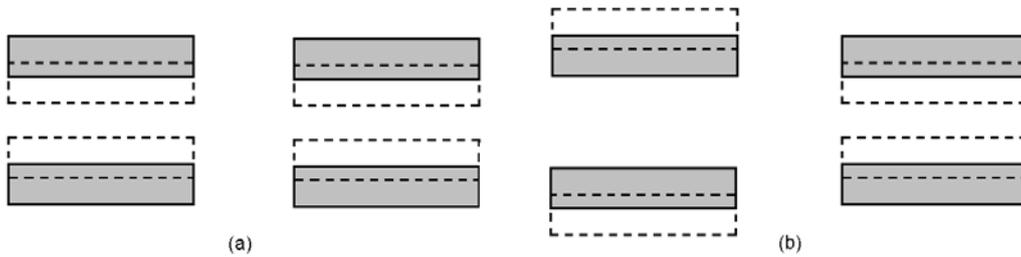


Fig. 5. Bounce of the bogies:
(a) symmetrical bounce; (b) antisymmetrical bounce.

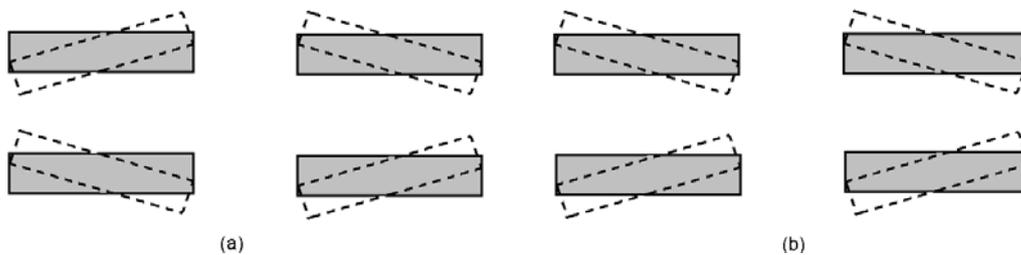


Fig. 6. Pitch of the bogies:
(a) symmetrical pitch; (b) antisymmetrical pitch.

The bounce and pitch vibrations of the bogies are transmitted via the secondary suspension to the carbody, which will also have symmetrical and antisymmetrical movements. The modes of symmetrical vibration are the carbody bounce and the front natural bending mode (two-node) (fig. 7), and the antisymmetrical vibration modes are the pitch and the rear natural bending mode (three-node) (fig.8). The carbody vibration in a random point is an overlapping of the four vibration modes.

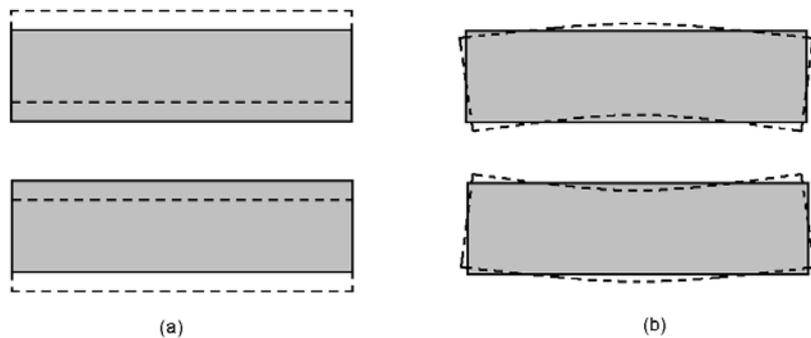


Fig. 7. The carbody symmetrical vibration modes:
(a) bounce; (b) two-node bending.

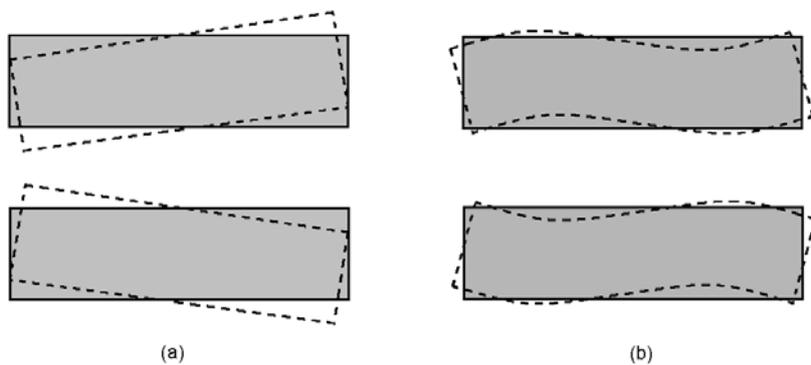


Fig. 8. The carbody antisymmetrical vibration modes:
(a) pitch; (b) three-node bending.

As for the mechanism that excites the rebound movements of the bogies, it should be mentioned that in the modern constructions, which provides a partial decoupling of the vibrations, they are not due to the pitch movements of the axles plans, but to the carbody bending vibrations. Similarly, the axles plans feature rebound movements, for which the excitation mode is different than of the bounce and pitch movements, in the sense that the excitation does not come straight from the track's irregularities, but via the vibration modes of the bogies (rebound and pitch).

The figure 9 includes the above and it is shown that the bending vibrations are transmitted from the carbody via the taking over system of the longitudinal forces to

the bogies, hence exciting their rebound movements. From here, the vibrations are conveyed through the elements of the longitudinal steering to the axles and generate rebound movements of their plans. The same thing happens with the rebound excitation of the axles plans, due to the pitch movement of the bogies.

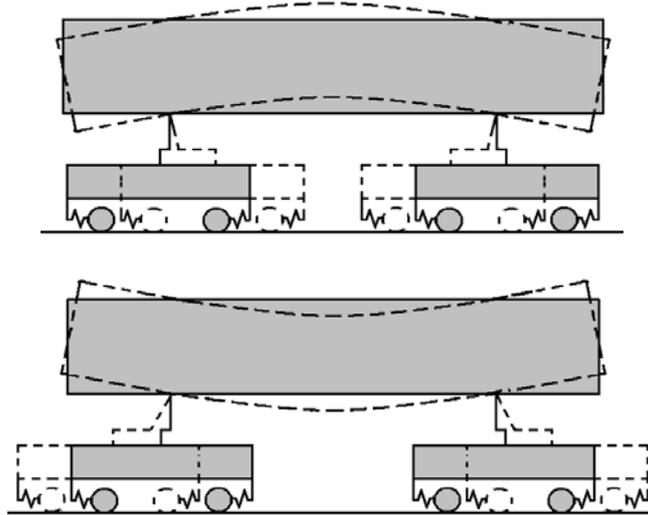


Fig. 9. The excitation mechanism of the rebound movements in bogies and axle plans.

The figure 10 (a) shows the symmetrical rebound of the bogies, defined by the fact that the bogies draw near and clear away simultaneously from the vehicle vertical symmetry axis. On the contrary, the simultaneous movement of the bogies occurs on either side of the vertical symmetry axis of the vehicle during the antisymmetrical rebound. (fig. 10, b).

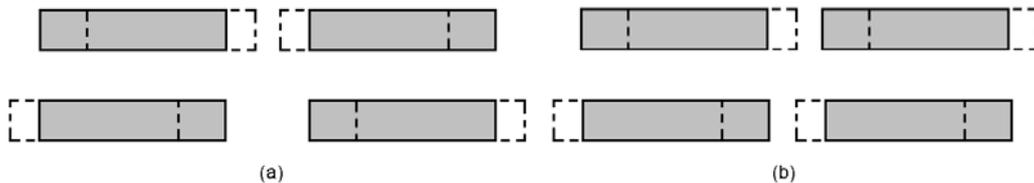


Fig. 10. The bogies rebound:
(a) symmetrical rebound; (b) antisymmetrical rebound

Upon using a similar reasoning with the above, the rebound shapes of the axle plans can be established, namely the symmetrical and antisymmetrical rebound (see fig. 11). For both types of movement, the axles in a bogie feature longitudinal travelling in phase at the same amplitude. What is different is that the symmetrical rebound of the axles in a bogie are in antiphase compared to the other bogie, while all the axles are in phase during the antisymmetrical rebound.

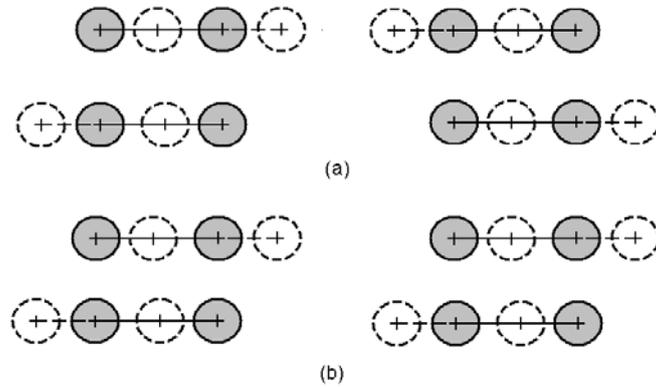


Fig. 11. The rebound of the axles plans:
 (a) symmetrical rebound; (b) antisymmetrical rebound.

The symmetrical and antisymmetrical movement modes of the axles plans, as well as the corresponding vehicle vibration modes can be pointed up at in an appropriate processing of the vehicle’s movement equations, as seen in the next section.

4. EQUATIONS OF THE VEHICLE SYMMETRICAL AND ANTISYMMETRICAL VIBRATION MODES

The vibrations of the vehicle presented in fig. 1 can be described by means of a 18 coupled equations system. Following a proper selection of the coordinates corresponding to the vibration symmetrical and antisymmetrical modes (Table 2), this system can be decomposed into two independent system, 8 equations each, where these systems detail the vehicle symmetrical and antisymmetrical vibration modes. Similarly, two more decoupled movement equations are obtained, specific to the free vibrations condition, which define the relative movements on the longitudinal direction between the axles of each bogie.

Likewise, based on the symmetry and antisymmetry properties of the natural functions in the front and rear modes of carbody vertical bending $X_2(x)$ și $X_3(x)$, the following notations are being proposed

$$X_2(l_1) = X_2(l_2) = \varepsilon^+; \quad X_3(l_1) = -X_3(l_2) = \varepsilon^-; \tag{10}$$

$$\frac{dX_2(l_1)}{dx} = -\frac{dX_2(l_2)}{dx} = \lambda^+; \quad \frac{dX_3(l_1)}{dx} = \frac{dX_3(l_2)}{dx} = \lambda^-. \tag{11}$$

While using the notations abov, the two systems that describe the vehicle symmetrical and antisymmetrical vibration modes are written under the matrix-type form

$$\mathbf{M}^+ \ddot{\mathbf{p}}^+ + \mathbf{C}^+ \dot{\mathbf{p}}^+ + \mathbf{K}^+ \mathbf{p}^+ = \mathbf{H}^+; \quad (12)$$

$$\mathbf{M}^- \ddot{\mathbf{p}}^- + \mathbf{C}^- \dot{\mathbf{p}}^- + \mathbf{K}^- \mathbf{p}^- = \mathbf{H}^-, \quad (13)$$

where $\ddot{\mathbf{p}}^\pm$, $\dot{\mathbf{p}}^\pm$, \mathbf{p}^\pm are the vectors of the state variables, namely the acceleration, velocity and travelling.

The matrices \mathbf{M}^+ and \mathbf{M}^- represent the inertia matrices, as

$$\mathbf{M}^+ = \text{diag}(m_c, m_{m2}, m_b, J_b, m_b, m_o, m_o, m_o);$$

$$\mathbf{M}^- = \text{diag}(J_c, m_{m3}, m_b, J_b, m_b, m_o, m_o, m_o).$$

The damping matrix, noted by \mathbf{C}^+ and \mathbf{C}^- , can be written as

$$\mathbf{C}^+ = \begin{bmatrix} 4c_{zc} & 4c_{zc}\varepsilon^+ & -4c_{zc} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & -4c_{zc}\varepsilon^+ & 2C_2\lambda^+ & 4c_{xc}h_c\lambda^+ & 0 & 0 & 0 \\ -2c_{zc} & -2c_{zc}\varepsilon^+ & C_{zbc} & -4c_{zb} & 0 & 0 & 0 & 0 \\ 0 & C_2\lambda^+ & 0 & C_3 & C_4 & 0 & -4c_{zb}a_b & 4c_{xb}h_{b1} \\ 0 & 2c_{xc}h_c\lambda^+ & 0 & C_4 & C_{xbc} & 0 & 0 & -4c_{xb} \\ 0 & 0 & -2c_{zb} & 0 & 0 & 2c_{zb} & 0 & 0 \\ 0 & 0 & 0 & -2c_{zb}a_b & 0 & 0 & 2c_{zb} & 0 \\ 0 & 0 & 0 & 2c_{xb}h_{b1} & -2c_{xb} & 0 & 0 & 2c_{xb} \end{bmatrix}$$

$$\mathbf{C}^- = \begin{bmatrix} C_5 & C_6 & -4c_{zc}a_c & 2C_2 & 4c_{xc}h_c & 0 & 0 & 0 \\ C_6 & C_7 & -4c_{zc}\varepsilon^- & 2C_2\lambda^- & 4c_{xc}h_c\lambda^- & 0 & 0 & 0 \\ -2c_{zc}a_c & -2c_{zc}\varepsilon^- & C_{zbc} & 0 & 0 & -4c_{zb} & 0 & 0 \\ C_2 & C_2\lambda^- & 0 & C_3 & C_4 & 0 & -4c_{zb}a_b & 4c_{xb}h_{b1} \\ 2c_{xc}h_c & 2c_{xc}h_c\lambda^- & 0 & C_4 & C_{xbc} & 0 & 0 & -4c_{xb} \\ 0 & 0 & -2c_{zb} & 0 & 0 & 2c_{zb} & 0 & 0 \\ 0 & 0 & 0 & -2c_{zb}a_b & 0 & 0 & 2c_{zb} & 0 \\ 0 & 0 & 0 & 2c_{xb}h_{b1} & -2c_{xb} & 0 & 0 & 2c_{xb} \end{bmatrix}$$

by using the following notations:

$$C_1 = c_{m2} + 4c_{zc}(\varepsilon^+)^2 + 4(c_{xc}h_c^2 + c_{\theta c})(\lambda^+)^2; \quad C_2 = 2(c_{xc}h_ch_{b2} - c_{\theta c});$$

$$C_3 = 4c_{zb}a_b^2 + 4c_{xb}h_{b1}^2 + 2c_{xc}h_{b2}^2 + 2c_{\theta c}; \quad C_4 = -4c_{xb}h_{b1} + 2c_{xc}h_{b2}$$

$$C_5 = 4(c_{zc}a_c^2 + c_{xc}h_c^2 + c_{\theta c}); \quad C_6 = 4(c_{zc}a_c\varepsilon^- + c_{xc}h_c^2\lambda^- + c_{\theta c}\lambda^-)$$

$$C_7 = c_{m3} + 4c_{zc}(\varepsilon^-)^2 + 4(c_{xc}h_c^2 + c_{\theta c})(\lambda^-)^2;$$

$$C_{zbc} = 4c_{zb} + 2c_{zc}; \quad C_{xbc} = 4c_{xb} + 2c_{xc}.$$

The matrices \mathbf{K}^+ and \mathbf{K}^- represent the rigidity matrices, as

$$\mathbf{K}^+ = \begin{bmatrix} 4k_{zc} & 4k_{zc}\varepsilon^+ & -4k_{zc} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_1 & -4k_{zc}\varepsilon^+ & 2k_2\lambda^+ & 4k_{xc}h_c\lambda^+ & 0 & 0 & 0 \\ -2k_{zc} & -2k_{zc}\varepsilon^+ & K_{zbc} & -4k_{zb} & 0 & 0 & 0 & 0 \\ 0 & K_2\lambda^+ & 0 & K_3 & K_4 & 0 & -4k_{zb}a_b & 4k_{xb}h_{b1} \\ 0 & 2k_{xc}h_c\lambda^+ & 0 & K_4 & K_{xbc} & 0 & 0 & -4k_{xb} \\ 0 & 0 & -2k_{zb} & 0 & 0 & K_{zbH} & 0 & 0 \\ 0 & 0 & 0 & -2k_{zb}a_b & 0 & 0 & K_{zbH} & 0 \\ 0 & 0 & 0 & 2k_{xb}h_{b1} & -2k_{xb} & 0 & 0 & 2k_{xb} \end{bmatrix}$$

$$\mathbf{K}^- = \begin{bmatrix} K_5 & K_6 & -4k_{zc}a_c & 2K_2 & 4k_{xc}h_c & 0 & 0 & 0 \\ K_6 & K_7 & -4k_{zc}\varepsilon^- & 2K_2\lambda^- & 4k_{xc}h_c\lambda^- & 0 & 0 & 0 \\ -2k_{zc}a_c & -2k_{zc}\varepsilon^- & K_{zbc} & 0 & 0 & -4k_{zb} & 0 & 0 \\ K_2 & K_2\lambda^- & 0 & K_3 & K_4 & 0 & -4k_{zb}a_b & 4k_{xb}h_{b1} \\ 2k_{xc}h_c & 2k_{xc}h_c\lambda^- & 0 & K_4 & K_{xbc} & 0 & 0 & -4k_{xb} \\ 0 & 0 & -2k_{zb} & 0 & 0 & K_{zbH} & 0 & 0 \\ 0 & 0 & 0 & -2k_{zb}a_b & 0 & 0 & K_{zbH} & 0 \\ 0 & 0 & 0 & 2k_{xb}h_{b1} & -2k_{xb} & 0 & 0 & 2k_{xb} \end{bmatrix}$$

where $K_1 = k_{m2} + 4k_{zc}(\varepsilon^+)^2 + 4(k_{xc}h_c^2 + k_{\theta c})(\lambda^+)^2$; $K_2 = 2(k_{xc}h_ch_{b2} - k_{\theta c})$;

$$K_3 = 4k_{zb}a_b^2 + 4k_{xb}h_{b1}^2 + 2k_{xc}h_{b2}^2 + 2k_{\theta c}; \quad K_4 = -4k_{xb}h_{b1} + 2k_{xc}h_{b2}$$

$$K_5 = 4(k_{zc}a_c^2 + k_{xc}h_c^2 + k_{\theta c}); \quad K_6 = 4(k_{zc}a_c\varepsilon^- + k_{xc}h_c^2\lambda^- + k_{\theta c}\lambda^-)$$

$$K_7 = k_{m3} + 4k_{zc}(\varepsilon^-)^2 + 4(k_{xc}h_c^2 + k_{\theta c})(\lambda^-)^2;$$

$$K_{zbc} = 4k_{zb} + 2k_{zc}; \quad K_{xbc} = 4k_{xb} + 2k_{xc}; \quad K_{zbH} = 2(k_{zb} + k_H);$$

Table 2. The coordinates of the symmetrical and antisymmetrical vibration modes of the vehicle.

Vibration modes	Symmetrical vibration modes	Antisymmetrical vibration modes
Carbody bounce	$p_1^+ = z_c$	-
Carbody pitch	-	$p_1^- = \theta_c$
Carbody bending	$p_2^+ = T_2$	$p_2^- = T_3$
Bogies bounce	$p_3^+ = \frac{z_{b1} + z_{b2}}{2}$	$p_3^- = \frac{z_{b1} - z_{b2}}{2}$
Bogies pitch	$p_4^+ = \frac{\theta_{b1} - \theta_{b2}}{2}$	$p_4^- = \frac{\theta_{b1} + \theta_{b2}}{2}$
Bogies rebound	$p_5^+ = \frac{x_{b1} - x_{b2}}{2}$	$p_5^- = \frac{x_{b1} + x_{b2}}{2}$
Bounce of the axles plans	$p_6^+ = \frac{z_{o1} + z_{o2} + z_{o3} + z_{o4}}{4}$	$p_6^- = \frac{z_{o1} + z_{o2} - z_{o3} - z_{o4}}{4}$
Pitch of the axles plans	$p_7^+ = \frac{z_{o1} - z_{o2} - z_{o3} + z_{o4}}{4}$	$p_7^- = \frac{z_{o1} - z_{o2} + z_{o3} - z_{o4}}{4}$
Rebound of the axles plans	$p_8^+ = \frac{x_{o1} + x_{o2} - x_{o3} - x_{o4}}{4}$	$p_8^- = \frac{x_{o1} + x_{o2} + x_{o3} + x_{o4}}{4}$

In the inertia matrices, of damping and rigidity that correspond to the two systems, terms such as m_{mn} , c_{mn} , k_{mn} (with $n = 2, 3$) can be found, and they represent the rigidity, damping and the modal mass, as case may be [6].

In the end, \mathbf{N}^+ și \mathbf{N}^- stand for the vectors of the heterogeneous terms, namely

$$\mathbf{N}^\pm = [0 \ 0 \ 0 \ 0 \ 0 \ 2k_H \eta_1^\pm \ 2k_H \eta_2^\pm \ 0]^\top,$$

where there are included symmetrical excitation modes and the antisymmetrical excitation modes brought about by the rolling track's irregularities,

$$\eta_1^\pm = \frac{\eta_1 + \eta_2 \pm \eta_3 \pm \eta_4}{4}; \quad \eta_2^\pm = \frac{\eta_1 - \eta_2 \mp \eta_3 \pm \eta_4}{4}. \quad (14)$$

5. CONCLUSIONS

The vibrations of the railway vehicles are the result of the interaction between the vehicle and the rolling track. The geometrical irregularities of the track, defects and discontinuities of the rolling surfaces in rails are major causes of the vibrations at the railway vehicles. The construction particularities turn the railway vehicle into a

complex oscillating system, with oscillating movements that occur in both vertical and horizontal plans, as translation and rotation movements, independent or coupled.

The vibration modes of the railway vehicles reunite the simple modes of vibration – rigid and complex, coming from the elasticity characteristics of the vehicle's suspended masses. These vibration modes are found in the form of symmetrical and antisymmetrical modes.

The paper focuses on the vibration symmetrical and antisymmetrical modes in a vertical plan of the vehicle's suspended masses and their excitation mechanism. This is the reason why the starting point is in the symmetrical and antisymmetrical movements, of bounce and pitch, of the axles plans caused by the rolling track irregularities. These movements are transmitted via the primary suspension for bogies, thus exciting their symmetrical and antisymmetrical modes of pitch and bounce. Further on, the bounce and pitch vibrations of the bogies are transmitted via the elements of the secondary suspension, and so the symmetrical and antisymmetrical modes of bounce, pitch and bending in the carbody are being excited. On their end, the carbody bending vibrations are transmitted through the system of taking over of the longitudinal forces at the bogies, thus exciting their symmetrical and antisymmetrical rebound vibrations. Such vibrations will be transmitted further via the elements of longitudinal steering at the axles, then triggering their symmetrical and antisymmetrical rebound movements.

Following a convenient selection of the coordinates, the symmetrical and antisymmetrical movement modes of the axles plans, as well as the corresponding vibration modes of the vehicle are pointed out at in the vehicle movement equations. Solving and analysing the issues related to the railway vehicle vertical vibrations will become much easier when using the equations mentioned above.

REFERENCES

- [1]. **Sebeşan I., Mazilu, T.,** *Vibrations of the railway vehicles*, MatrixRom Publishing House, Bucharest, 2010, (Romanian language).
- [2]. **Carlbon, P.,** *Carbody and Passengers in Rail Vehicle Dynamics*, Doctoral Thesis, Division of Railway Technology, Department of Vehicle Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.
- [3]. **Sebeşan, I.,** *Dinamica vehiculelor feroviare (Dynamic of the railway vehicles)*, Matrix Rom Publishing House, Bucharest, 2011, (Romanian language).
- [4]. **Cheli, F., Corradi, R.,** *On rail vehicle vibrations induced by track unevenness: Analysis of the excitation mechanism*, Journal of Sound and Vibration, Vol. 330, 2011, pp. 3744–3765.
- [5]. **Diana, G., Cheli, F., Collina, A., Corradi, R., Melzi, S.,** *The development of a numerical model for railway vehicles comfort assessment through comparison with experimental measurements*, Vehicle System Dynamics, Vol. 38, no. 3, 2002, pp. 165- 183.
- [6]. **Dumitriu, M.,** *Influence of the vertical suspension on the vibration behavior in the railway vehicles*, Annals of the University of Petroşani, Mechanical Engineering, Vol. 13, 2011, pp. 35-50.
- [7]. **Dumitriu, M.,** *Evaluation of the comfort index in railway vehicles depending on the vertical suspension features*, Annals of Faculty Engineering Hunedoara – International Journal of Engineering, Tome XI, 2013, Fascicule 4, pp. 23-32.