PARAMETERS MODELLING FOR CYCLOIDAL MESHING GEAR TEETH

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Abstract: Although the models of finite elements have efficiency and versatility, in the analysis of the mechanical strains and tensions which appear in the case o meshing gear teeth, the variation of the contact force and the dynamical load in the process of meshing are usually neglected. We have shown aspects of the parameters change for pairs of cycloid gearwheels in the process of meshing in static and dynamic conditions, as well as the algorithm for the contact method of the finite element. For a certain rotation speed and torsion torque of the load, with the help of the computerized simulation method, we can determine the variation of the contact force on the side (lateral side) of the meshing gearwheels. The values for mechanical contact tensions can be calculated taking into consideration the elastic deformation of toothed mechanisms, the rigidity variation of the meshing teeth.

Keywords: dynamic contact, finite element method, gear meshing.

1. INTRODUCTION

Because of the complexity of the meshing teeth transmission mechanisms, the constant improvement requirements and increasing lifespan, a great interest arose in understanding the behavior of the transmission of these toothed mechanisms in many areas of engineering design as well as in the manufacturing process. The static and dynamic performances of the toothed mechanism transmission are influenced by many factors that include: geometrical parameters, fabrication errors, variations of the tooth strain and rotation speed, tooth deformation, tooth mechanism impact due to the meshing variation between one and two pairs of teeth, axel rigidity, assembling rigidity, etc. In theory, the system of meshing tooth mechanisms is usually modeled as a dynamic system described by linear or nonlinear functions. An analytic solution that answers through load effects, gear rigidity, rotations speeds as well as cushioning, could generally be obtained by calculating the meshing rigidity and cushioning coefficients for meshing toothed gears.

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Many research cases, using the method of the finite element for meshing toothed gears, have provided numerical solutions for load distribution in the case of contact for lateral sides of the tooth flank, as well as gear tooth deformation. Trying to understand the dynamics of mechanisms requires researching the variation and distribution of contact loads, the impact of meshing gears as well as tensions resulted with the passing of time. As shown in the following paragraphs analyses have been made that define the simulation of meshing toothed gears.

Then the algorithm of the finite static contact element for the meshing system with toothed gearwheels is presented as a solution for the contact force for the gearwheel tooth in the meshing process.

In this algorithm the deformation conditions, the contact force an the Columbian friction force have the purpose of describing the static meshing conditions for the teeth of toothed gears, while the Cholsky method as well as Newmark's algorithm have been used for the static force. To improve the numerical analysis of the solution, an equation depending on the gears' teeth was established and a graphic program for finite elements was used. With a certain rotation speed and torsion torque of the finite element we obtain the variation of the contact force on the toothed gear's tooth, throughout the process.

2. MODELING THE PARAMETERS FOR THE MESHING TEETH PAIRS

We establish a number of models for finite contact elements in the case of toothed gears to assess the variation of the contact force in the meshing process and to find the solutions for each contact point along the action line.

These models should respect the geometrical principle. As shown in figure 1, the global Cartesian coordinates (0 – X ,Y , Z) of the meshing tooth pairs are the same as the local coordinates of the lead wheel (2) (0 – X_2 ,Y_2 , Z_2) while the coordinates (0 – X_1 ,Y_1 ,Z_1) are local coordinates of the leading wheel (1). The local contact coordinates of the meshing points are (0 – *n*, *t*, *s*).

The coordinates of the meshing point "*i*" can be written:

$$\begin{cases} x_{ik} = cos\,\alpha \cdot B_{ip} \\ y_{ik} = \dfrac{d_k}{2} \pm sin\,\alpha \cdot B_{ip} \end{cases} \quad (k = 1, 2) \tag{1}$$

$$B_{ip} = \frac{1}{2}\left[-d_k \sin\alpha + \left(d_k^2 \sin^2\alpha - d_k^2 + 4\,r_{ik}^2 \right)^{\frac{1}{2}} \right] \tag{2}$$

where: d_k – dividing diameter of the two gearwheels; α – pressure angle; r_{ik} – meshing point "*i*" radius from the tooth flank of the two meshing wheels; B_{ip} – distance between the meshing point "*i*" and the meshing pole "*p*" from the meshing line.

In the meshing process the initial contact appears in “A” and with the spin of the gearwheel the contact point will move along the meshing line. To establish the contact of the models of the finite element in the meshing process, contact point “*i*” and the matching contact point “*j*” for another teeth pair can be determined if the model of the finite element represents the contact of two teeth pairs.

After calculating the two contact points “*i*”,”*j*” the program allows the discretization with finite elements, elements that can be generated parametrically or automatically. The dividing numbers for finite elements of the teeth can be random.
The coordinates of the points of the finite elements for other sections parallel with the transversal section are calculated by expanding the discretization of fixed elements along the axial direction, of the discredited gearwheel.

The situation shown above is valid for modeling gearwheels with teeth in an involute profile. As follows in fig.1 and fig.2 we present a sketch of a meshing system for cycloid gearwheels and their coordinates.

In fig. 2 we show the modeling with finite elements for one single pair of teeth in contact, corresponding to different positions on the meshing line in the case of gearwheels with teeth with cycloid profile.

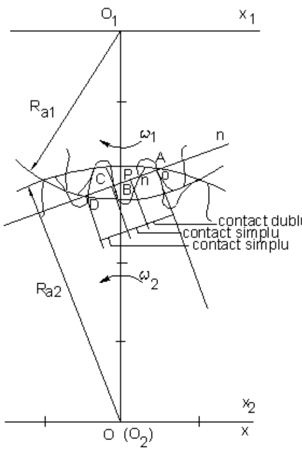


Fig. 1. Cartesian coordinates of gearwheels

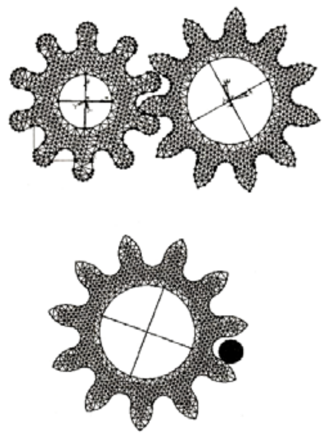


Fig. 2. Finite elements modeling of the cycloidal gearwheels

3. STATIC, DYNAMIC AND CINEMATIC CONDITIONS FOR THE MESHING TEETH PAIRS

In the meshing process, contact between teeth flank changes as position from one pair of teeth to two pairs of teeth. Certain static, dynamic and cinematic conditions for the discretized pairs of teeth can be observed through local contact coordinates (0 – *n*, *t*, *s*). Presuming that no separation appears at the contact point “*i*” corresponding to two knots in contact for each tooth, the values of motion and speed for two knots in contact should be the same on the normal direction "n" in the local coordinates system, the law of Columbian friction acts on the tangent directions (t,s).

This is why the meshing conditions for the teeth, both static, dynamic and cinematic in the time unit Δt, along the movement can be represented by:

$$\begin{cases} R_{1n}^{t+\Delta t} = -R_{2n}^{t+\Delta t} \\ R_{1t}^{t+\Delta t} = -R_{2t}^{t+\Delta t} = \mu\, sin^{t+\Delta t}\, \beta \left| R_{1n}^{t+\Delta t} \right| \\ R_{1s}^{t+\Delta t} = -R_{2s}^{t+\Delta t} = \mu\, cos^{t+\Delta t}\, \beta \left| R_{1n}^{t+\Delta t} \right| \\ \dot{U}_{1n}^{t+\Delta t} = \dot{U}_{2n}^{t+\Delta t} \\ U_{1n}^{t+\Delta t} = U_{2n}^{t+\Delta t} \end{cases} \tag{3}$$

where: $R_{1e}^{t+\Delta t}$, $R_{2e}^{t+\Delta t}$, ($e = n, t, s$) are the contact forces for each tooth on n, t, s – directions of the coordinates axes of the local contact system; μ – Columbian friction coefficient; $\dot{U}_{1n}^{t+\Delta t}$, $\dot{U}_{2n}^{t+\Delta t}$ and $U_{1n}^{t+\Delta t}$, $U_{2n}^{t+\Delta t}$ are the speeds and movements of the normal direction of the local contact coordinates; β – relative slide direction of the meshing teeth pair.

For the static contact analysis the time variation Δ*t* and the speed are neglected in formulas 3÷6. Considering that the separation could take place in the moment of meshing impact of different teeth pairs and because of manufacturing errors of the teeth flanks and of the vibromechanical resonance phenomenon, in the meshing process are used as detection criteria in interactive calculus for the static and dynamic contact analysis:

$$\begin{cases} R_{1n}^{t+\Delta t} = -R_{2n}^{t+\Delta t} > 0 \\ \delta_n^{t+\Delta t} = U_{2n}^{t+\Delta t} - U_{1n}^{t+\Delta t} + \delta_{0n}^{t} > 0 \end{cases} \tag{4}$$

Separation

$$\begin{cases} R_{1n}^{t+\Delta t} = -R_{1n}^{t+\Delta t} \le 0 \\ \delta_t^{t+\Delta t} = U_{2t}^{t+\Delta t} - U_{1t}^{t+\Delta t} + \delta_{0t}^{t} > 0 \end{cases} \tag{5}$$

Relative slide

$$\begin{cases} R_{1n}^{t+\Delta t} = -R_{2n}^{t+\Delta t} \le 0 \\ \delta_t^{t+\Delta t} = U_{2t}^{t+\Delta t} - U_{1t}^{t+\Delta t} + \delta_{0t}^{t} < 0 \end{cases} \tag{6}$$

Where $\delta_e^{t+\Delta t}$, δ_e^{t} ($e = n, t$) are the relative movements along directions n, t.

4. FLEXIBILITY FORMULAS FOR THE METHOD OF THE FINITE CONTACT ELEMENT

Although the two parts of the leading and lead meshing gears can satisfy the rigidity situation of the finite element (7) in the case of static contact analysis for simulating the dynamic answer of the meshing gear teeth in time many factors such as elastic deformation, inertial force, rotation speed variation due to changes in meshing teeth pair influence the numeric results. The dynamic equation for the method of the finite element in the case of meshing teeth pair neglecting the cushioning forces in a moment in time $t + \Delta t$ comes out of the formula:

$$K_i U_i = P_i + R_i \quad , \quad (i = 1, 2) \tag{7}$$

$$M_i^{t+\Delta t} \ddot{U}_i^{t+\Delta t} + K_i^{t+\Delta t} U_i^{t+\Delta t} = P_i^{t+\Delta t} + R_i^{t+\Delta t} \quad , \quad (i = 1, 2) \tag{8}$$

Where K_i, $K_i^{t+\Delta t}$ (i= 1, 2) are the rigidity matrix' for each meshing wheel; $M_i^{t+\Delta t}$ – the mass matrix for each meshing matrix; P_i, $P_i^{t+\Delta t}$ - the load vectors caused by the torsion moment of the leading wheel and the reaction torsion moment of the lead wheel; R_i, $R_i^{t+\Delta t}$ - contact forces in the contact point of the meshing teeth; U_i, $U_i^{t+\Delta t}$ the displacement vectors of each meshing gear; $\ddot{U}_i^{t+\Delta t}$ - acceleration vectors for each meshing wheels

The direct integration algorithm in the case of the finite element method is used as a solution of the dynamic equation of meshing gears with teeth based upon two hypothesis': 1) the numerical solution of the dynamic equation is obtained on the discreet time interval Δt; 2) the variation of movement, speed and acceleration for each gear mesh at the random time interval Δt is presumed to have linear acceleration relations. With these two suppositions, the dynamic behavior solution for the meshing process can be obtained by solving a number of static equations in different contact points of the meshing teeth pairs along the meshing line in the given time interval.

In the following lines the Newmark's integration algorithm is used, with the next linear acceleration relation:

$$\dot{U}_i^{t+\Delta t} = \dot{U}_i^{t} + \left[(1-\delta)\ddot{U}_i^{t} + \alpha \ddot{U}^{t+\Delta t} \right] \Delta t \quad , \quad (i = 1, 2) \tag{9}$$

$$U_i^{t+\Delta t} = U_i^t + \dot{U}_i^{t} \Delta t + \left[\left(\frac{1}{2} - \alpha \right) \ddot{U}_i^{t} + \alpha \ddot{U}^{t+\Delta t} \right] \Delta t^2 \quad , \quad (i = 1, 2) \tag{10}$$

Where $U_i^{t+\Delta t}, \dot{U}_i^{t}$ – are speed vectors for each gearwheel that meshes at the time $t + \Delta t$, t; δ, α – are parameters of Newmark's integration algorithm.

If we replace equation (9) and (10) in equation (8) the dynamic equation for the meshing gears at the time $t + \Delta t$ can be written as the following equation of similar shape as the rigidity equation in the static analysis of the finite element.

$$\hat{K}_i^{t+\Delta t} \cdot U_i^{t+\Delta t} = \hat{P}_i^{t+\Delta t} + R_i^{t+\Delta t} \quad , \quad (i = 1, 2) \tag{11}$$

$$\hat{K}_i^{t+\Delta t} = \frac{1}{\alpha\,\Delta t^2} M_i + K_i^{t+\Delta t} \quad , \quad (i = 1, 2) \tag{12}$$

$$\hat{P}_i^{t+\Delta t} = P_i^{t+\Delta t} + \frac{1}{\alpha\,\Delta t^2} M_i U_i^t + \frac{1}{\alpha\,\Delta t} M_i \dot{U}_i^t + \left(\frac{1}{2\,\alpha} - 1\right) M_i \ddot{U}_i^t \quad , \quad (i = 1, 2) \tag{13}$$

where $\hat{K}_i^{t+\Delta t}$ and $\hat{P}_i^{t+\Delta t}$ are respectively, the effective rigidity matrix and the effective load vector in the direct integration algorithm.

Using the dynamic, cinematic and static conditions for the static analysis of contact points for the above mentioned mashing gearwheel pairs, the dynamic contact force $R_i^{t+\Delta t}$ (or the static contact force R_i) and the movement $U_i^{t+\Delta t}$ (or U_i) can be obtained by solving the effective rigidity equation (11) (or equation (7) for static analysis) and then the speed and movement vectors can be also obtained through (9) and (10).

To improve the calculus with iterative finite element of the contact analysis we use a flexibility formula of the contact finite element in which the static, dynamic and cinematic conditions are contained by the matrix' sub-coefficients. By developing a modified Choleshy scheme, for elimination by decomposing the solution matrix of the linear equation and calculating the corresponding changes for the sub-coefficient matrixes', the flexibility matrix equation at the meshing teeth's contact point can be derived:

$$\hat{F}^{t+\Delta t} \cdot R_i^{t+\Delta t} = \delta^{t+\Delta t} - \hat{\delta}_p^{t+\Delta t} \tag{14}$$

$$\hat{F}^{t+\Delta t} = D^{t+\Delta t} - C_1^{t+\Delta t}\, D_1^{t+\Delta t} - C_2^{t+\Delta t}\, D_2^{t+\Delta t} \tag{15}$$

$$\hat{\delta}_p^{t+\Delta t} = C_1^{t+\Delta t} \cdot \hat{P}_1^{t+\Delta t} + C_2^{t+\Delta t} \cdot \hat{P}_2^{t+\Delta t} \tag{16}$$

where : $\hat{F}^{t+\Delta t}$ is the effective flexibility matrix of the meshing contact point; $\delta^{t+\Delta t}$ – the values of the initial space at the contact points; $\hat{\delta}^{t+\Delta t}$ - relative movement due to exterior load and rotation speed variation; $C_1^{t+\Delta t}$, $C_2^{t+\Delta t}$, $D_1^{t+\Delta t}$, $D_2^{t+\Delta t}$ and $D^{t+\Delta t}$ - are sub-coefficient matrixes corresponding to the static, dynamic and cinematic conditions.

As soon as the flexibility matrix equation is obtained, the solution of the iterative calculus in the case of static or dynamic analysis for the meshing process is reduced to the contact points of the gearwheel teeth in such a way that a numeric simulation of the behavior in static and dynamic regime of the meshing teeth with three-dimensional modules of the finite element can be made.

5. STATIC CONTACT ANALYSIS PROGRAM FOR MESHING GEAR TEETH

According to the static contact analysis algorithm for meshing gear teeth shown above, a numeric modeling program is developed, including the parameters of meshing gearwheel teeth; static and cinematic definitions; iterative calculation of the static contact force and calculation of mechanical tensions for meshing gearwheel teeth.

Table 1. **Main parameters for gearwheels**

Number of teeth		Distance between axes, a	Module, m	Pressure angle	Coverage degree e	Transmission ratio	Tooth width	Rotation speed, rot/min	Torsion torque T1 (motor)	Friction coefficient, μ
z_1	z_2	334,215	31,83	20^o	1	$\frac{10}{11}$	80,75	5,62	100	0,1
10	11									

In order to increase efficiency concerning the evaluation of the static behavior for meshing gearwheel teeth a post-processing is necessary to verify the meshing gears with a finite element, presenting the mechanical tensions and specific deformations, etc.

6. STATIC CONTACT ANALYSIS PROGRAM FOR MESHING GEAR TEETH

According to the static contact analysis algorithm for meshing gear teeth shown above a part of the finite element graphic program is shown, in which are included: the modeling parameters for the teeth of the cycloidal meshing gearwheels; static definitions; iterative calculation of static forces and calculation of deformation and tensions.

The main stages for solving contact problems are shown in figure 3 as follows:

- introducing entry data which consists in making the meshing drawing (or import of a DXF file from ACAD); introducing the mechanical characteristics (elasticity module, Poisson coefficient, specific weight, friction coefficient, etc.); discretization of the meshing gears, applying

movement restrictions, introducing efforts;
- the pre-processing procedure which helps find the eventual flaws consists of presenting the contour of the gearwheel, presenting the discretization, presenting the efforts and applied restrictions;
- calculating the rigidity matrix for each element;
- assembling the rigidity matrix;
- solving the movement equation system;
- calculating deformation and tension within elements;
- exit data that consist in a file of numeric results and a file of graphical results following the post processing procedure (presenting the discretization, presenting the tension state after different theories, presenting specific deformations, presenting movement, etc.).

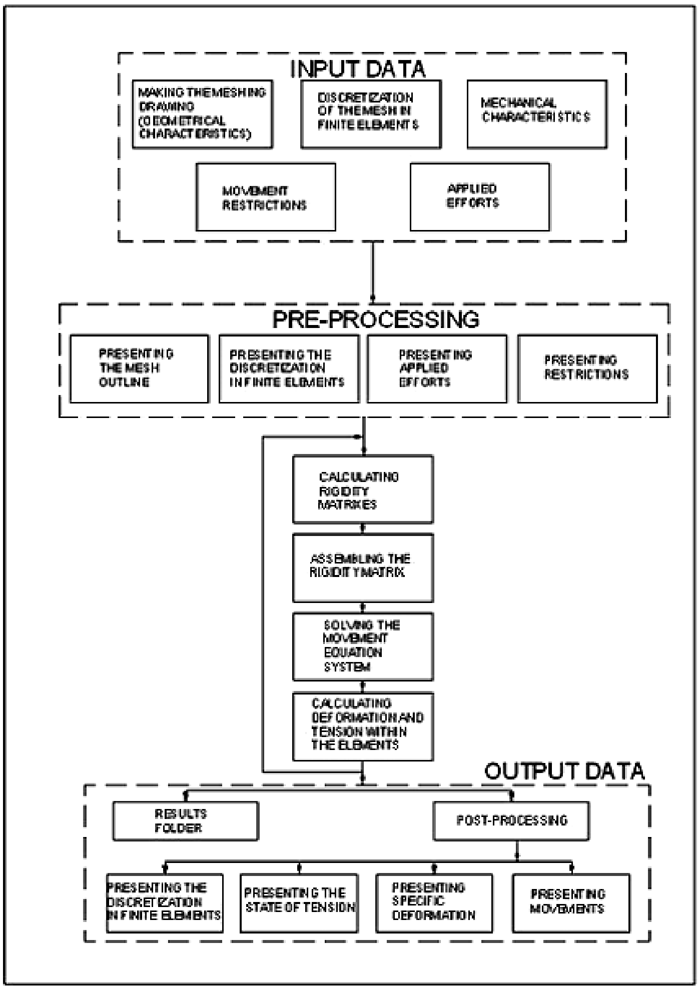


Fig. 3: The main stages for solving contact problems

7. MODELLING CYCLOIDAL MESHING GEARWHEELS WITH THE FINITE ELEMENT METHOD

In order to make the model for cycloidal meshing gearwheels, starting from the execution drawing the two wheels in contact were made as two surfaces formed by 242 points between which 214 straight lines have been drawn. In figure 2 is shown the discretization mode for gearwheels through plane triangular elements with a constant thickness g = 75 mm.

The calculus model has 2113 knots and 3658 triangular elements with three knots, modeling which generated 6937 equations. The idealized model elements of the defined structure on discredited knots are called finite elements. For the analysis of tension states and deformations the analysis step was 1/10 of the meshing angle: figures A.1…A.10…B.1…B.11.

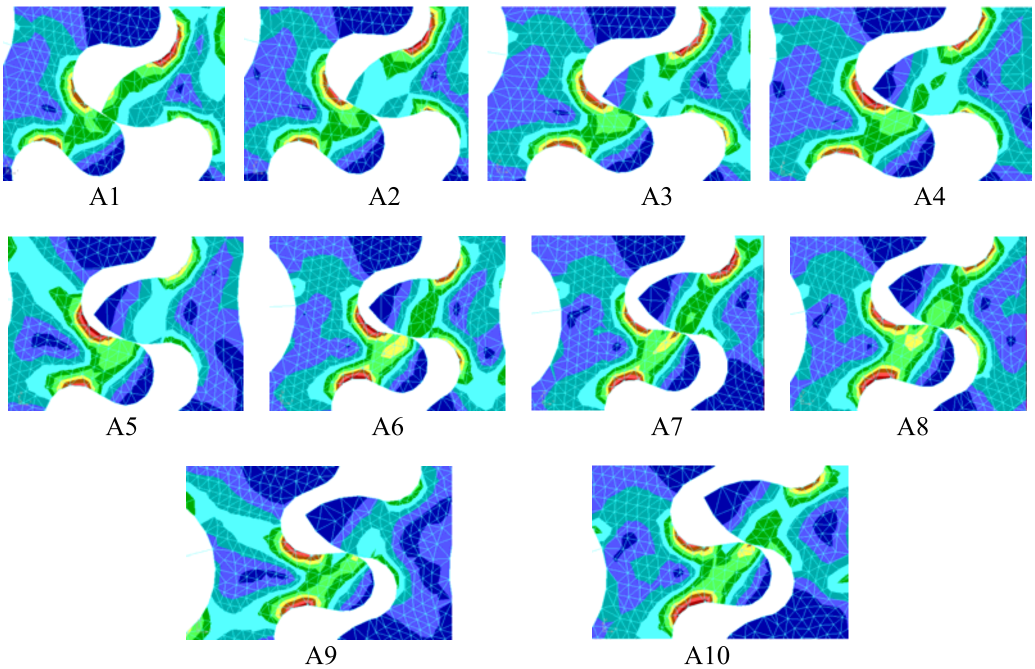


Figure 4: State of tension between the leading and the lead wheels in the dividing steps

For the calculus of the tension distribution an exterior load with a unitary moment was used.

8. RESULTS AND DISCUSSIONS

To validate the computer simulation method including the previously analyzed aspects we present the static contact analysis for the teeth of cycloidal meshing gearwheels in the meshing process with the algorithm of the finite element method.

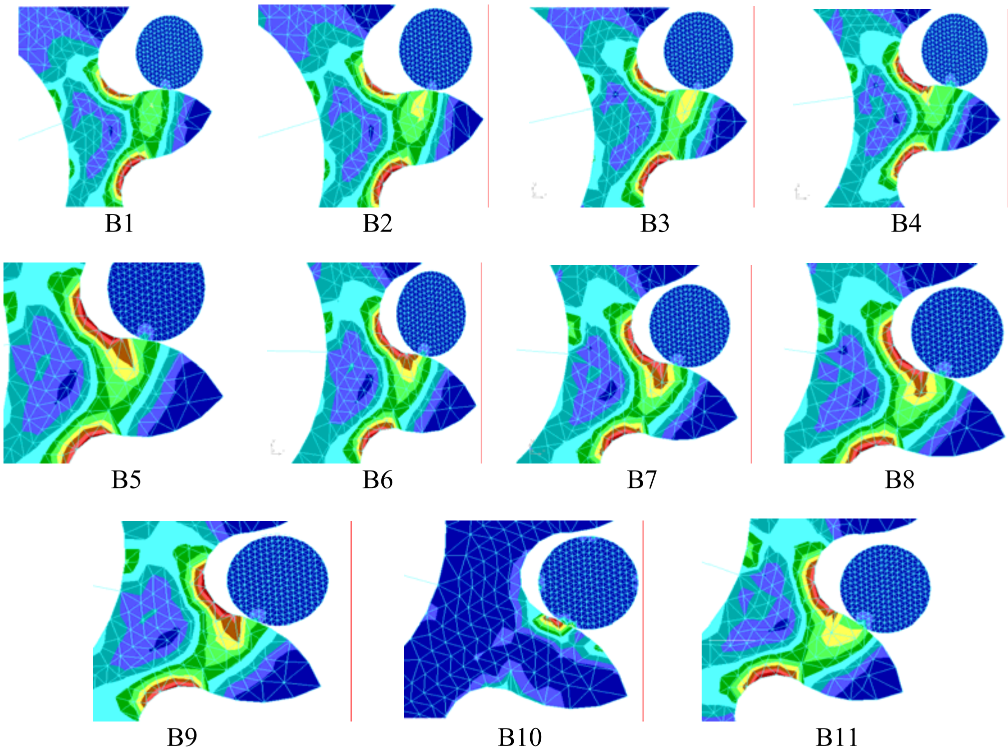


Fig. 5. State of tension on the lead wheel at the meshing with the rack pin

The geometric parameter, torsion torque of the load for this numerical analysis is shown in table 1.

Using the static contact analysis program we obtain the static tension distribution on the teeth profile and the pin in the contact area, as shown in figures A.1…A.10 and B.1…B.11.

Figures A.1…A.10 shows the tension state after the theory of Von Mieses capitalizing on the 10 mesh dividing steps between the leading wheel and the lead wheel, the maximum tension on the lead wheel being 10,4 × 10^{-6} N/m^2 at the meshing entrance and at step 4 according to fig. A.4. the maximum tension at the tooth base on the leading wheel is 9.98 × 10^{-6} N/m^2.

Figures C1…C9 show the tension distribution on the lead wheel following the sequence of steps corresponding to each sectioning plane. Figures D1…D7 show the same tension distribution for the meshing leading wheel with the rack pin.

Figures B.1…B.11 shows the tension state after the Von Mieses' theory capitalizing on the 10 dividing steps of the mesh between the lead wheel and the pin, the maximum tension being 6.41 × 10^{-6} N/m^2. The tooth of the intermediate wheel was sectioned by eight planes. Figures D.1…D.7 show the tension distribution following the sequence of steps corresponding to each sectioning plane. The distances between the sectioning lines are considered of unitary value.

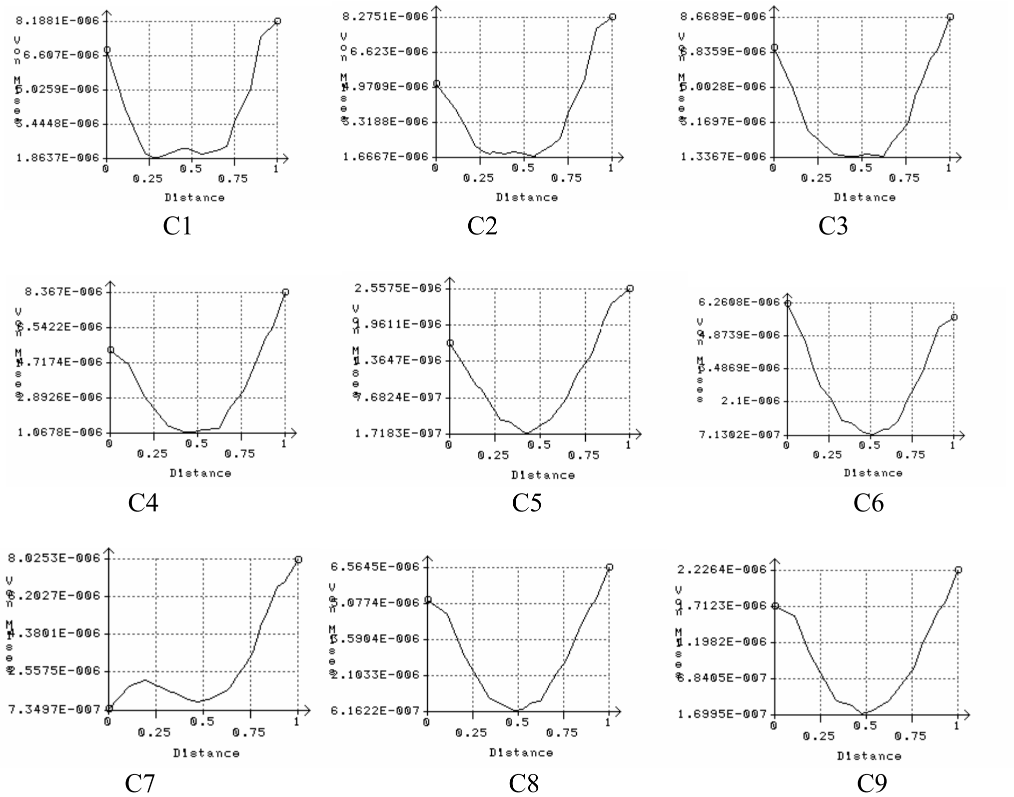


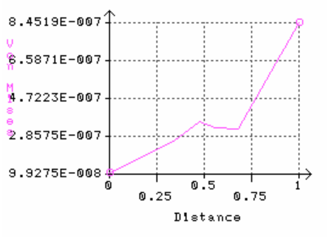
Figure 6: Tension distribution for the lead and leading wheel

The bending tension at the tooth base for the lead and leading wheels reaches the maximum when the contact point on the tooth flank is situated on the tooth top in the area with one single tooth pair in contact, the cause being the integrated effects of the changes in normal contact forces as well as the distance from the contact point to the tooth base on its flank.

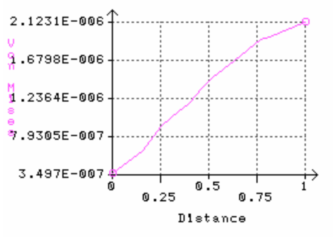
9. CONCLUSION

Using the developed computer simulation procedure, the static and dynamic contact force distribution and stress of finite element model on different contact point along the line of action have been obtained, which can be used to evaluate the static and dynamic characteristics of meshing gear teeth in the process of engagement.

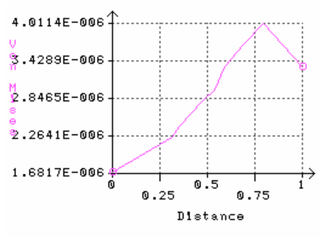
In this research, static and dynamic behavior of meshing gear teeth with the consideration of the geometric parameters, elastic deformation, and stiffness variation of meshing tooth pairs, rotating speed and driving torque are analyzed. And the influence of other factors such as manufacturing error can be reached if the tooth flank can be precisely determined for generating respective finite element models.



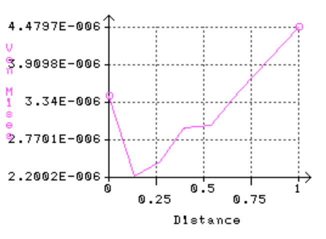
D1



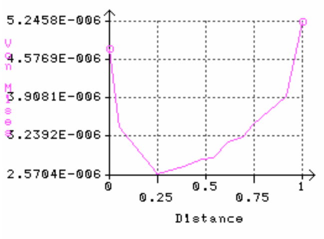
D2



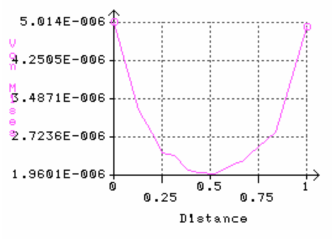
D3



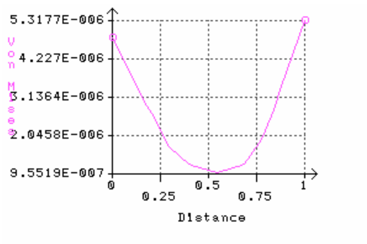
D4



D5



D6



D7

Fig.7. Tension distribution for the lead wheel and the rack pin

REFERENCES

[1]. **Zamfir, V., Dimirache, G.**, *The Study of the "Cogwheel-Rack" Gearing on Mining Combines' Advance Systems*, National Symposium for Mechanisms and Mechanical Transmissions, Timisoara, 1992 (Romanian language);

[2]. **Petrilan, T.**, *The Finite Element Method and Applications*, Academical Publishing, Bucuresti, 1987, (Romanian language);

[3]. **Hengan, Qu, Hui Long, Rajbalendra,** *Computer Simulation for Meshing Gear Teeth with Static and Dynamic Contact Finite Element Method*, Departament of Mechanical Engineering China